Topic: Infinite limits (vertical asymptotes)

Question: Find the limit.

$$\lim_{x \to \pi} \cot x$$

Answer choices:

A 0

B 1

C ∞

D Does not exist (DNE)

© Krista King Math

Solution: D

The function approaches $-\infty$ as x approaches π from the left and ∞ as x approaches π from the right. Therefore, since these values are unequal, the limit as x approaches π does not exist for this function.

© Krista King Math

Topic: Infinite limits (vertical asymptotes)

Question: Find the limit.

$$\lim_{x \to 1} \frac{1}{(x-1)^2}$$

Answer choices:

A 0

B 1

C ∞

D Does not exist (DNE)

© Krista King Math

Solution: C and D

Choice C is correct because the function approaches ∞ as x approaches 1 from the left and from the right. When the function exhibits unbounded behavior, we say the limit is either equal to $+\infty$ or $-\infty$ in order to describe whether the function is unbounded in the positive direction or the negative direction, even though technically the limit does not exist.

Choice D is correct because the function increases without bound as x approaches 1, and so, being an infinite limit, the limit technically does not exist.

© Krista King Math

Topic: Infinite limits (vertical asymptotes)

Question: Find the value of the limit.

$$\lim_{x \to \frac{\pi}{2}} \tan(x)$$

Answer choices:

A $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = -\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan(x) = +\infty$

B $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = +\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$

C $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = -\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$

D $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = +\infty$ $\lim_{x \to \frac{\pi}{2}^+} \tan(x) = +\infty$

© Krista King Math

Solution: B

First we'll rewrite the given function.

$$\lim_{x \to \frac{\pi}{2}} \tan(x)$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin(x)}{\cos(x)}$$

Evaluate the limit.

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)}$$

$$\frac{1}{0}$$

This answer does not exist so instead we can try to find the limit by using numbers close to the limit. $\pi/2 \approx 1.57$, so we'll need to look at values on either side of that.

From the negative direction, we can try some values that get closer to the limit.

If we substitute $x = 1.50001$, the answer is 14.096.

If we substitute $x = 1.5001$, the answer is 14.11.

If we substitute $x = 1.501$, the answer is 14.3.

If we substitute $x = 1.51$, the answer is 16.4.

© Krista King Math

From this direction, we can see that the function is heading towards positive infinity.

From the positive direction, we can try some values that get closer to the limit.

If we substitute $x = 1.61$, the answer is 16.4.

If we substitute $x = 1.601$, the answer is -33.1.

If we substitute $x = 1.6001$, the answer is -34.1.

If we substitute $x = 1.60001$, the answer is -34.2.

For this direction, we can see that the function is heading towards negative infinity.

This means that $\lim_{x \to \frac{\pi}{2}^-} \tan(x) = +\infty$ and $\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$.

© Krista King Math